FINAL REPORT

ACTIVE CHAOTIC FLOWS, DETERMINISTIC MODELING, AND COMMUNICATION WITH CHAOS

OFFICE OF NAVAL RESEARCH

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13. ABSTRACT (*Maximum 200 words*)

This is the final report on this project. The objectives were threefold:
1) to study the chemical and biological activity in environmental flows often involving larger particles which are influenced by inertia, buoyance, Stokes' drag, and gravity forces;
2) to establish to what extent a natural chaotic system can be modeled deterministically; and
3) to demonstrate theoretically and experimentally that we can encode a message in a power oscillator (source) which is then transmitted through some communication channel (e.g. optical fiber, microwave antenna) to be received by a decoder which translates the message back to its original form, the goal being to use a method for synchronizing a chaotic oscillator to the incoming signal such that in-band noise is reduced.

14. SUBJECT TERMS

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I. TECHNICAL OBJECTIVES

The technical objectives of the contract are threefold. Firstly, the study of chemical and biological activity in environmental flows often involving larger particles which are influenced by inertia, buoyance, Stokes' drag, and gravity forces. In this case, instead of the trajectories obeying a Hamilton's type system of equations, they obey a dissipative system of equations which display typical chaotic phenomena. In particular, with the proper boundary conditions, the concentration of particles in physical space can be restricted to a fractal chaotic attractor. Because the chaotic dynamics is fundamentally different, volume preserving in Hamiltonian dynamics before versus zero volume in dissipative dynamics now, one expects the active processes themselves to be also different. In other words, productivity of the process must reflect the competition between *emptying* through the dissipative character of the flow and *filling* due to the reaction itself. The enhancement in productivity results now from the fractal structure of the invariant attracting chaotic set.

Secondly, to establish to what extent a natural chaotic system can be modeled deterministically. More specifically, say one constructs a physical chaotic system in a laboratory, and one measures a trajectory. The question then is whether there is a trajectory of reasonable length from the mathematical model of the physical system that is close to the measured physical trajectory. We stress that subtleties and difficulties of numerical calculations of chaotic systems are not the issue here. The difficulty to model the natural process is a consequence of the inexactitude of the model given by the inevitable random disturbances and imperfections of the model such as various approximations used in the model-building process and intrinsic dynamical properties of the physical systems under consideration.

Thirdly, we have demonstrated theoretically and experimentally that we can encode a message in a power oscillator (source). This message is transmitted through some communication channel (e.g., optical fiber, microwave antenna) to be received by a decoder which translates the message back to its original form. Due to various kinds of noise, distortion and interference, the signal at the end of the channel is not quite the same as the signal

at its beginning. And here lies another fundamental problem in communication: how to transmit a message without distortion such that the signal received at the end of the channel is a faithful reproduction of the signal at the beginning of the channel. The major inherent difficulty is with in-band noise. If a filter is used to eliminate, even partially, this type of noise, the true signal itself will be affected since the filter operates in a frequency range that includes the signal. The objective then is to use a method for synchronizing a chaotic oscillator to the incoming signal such that in-band noise is reduced.

II. TECHNICAL APPROACH

The technical approach taken in each of the objectives are the following. Firstly, when the particle dynamics is Hamiltonian, as in the case of passive scalars, the emptying of the product along the unstable manifold of the chaotic saddle occurs if one has an open flow. On the other hand, if the dynamics is dissipative and possesses an attracting fractal set in physical space, this set plays the role of the unstable manifold in the Hamiltonian case, and the emptying occurs through the *contraction* of phase space volume in the neighborhood of the attracting set. Although the emptying mechanisms are different, the same type of competition between the emptying, now due to contraction, and the fattening-up by the reaction is still present. Thus, one sees active particles and products to be in the neighborhood of the attracting fractal set, this set being the *skeleton* of the reaction. Although most of the environmental flows are open flows, it is meaningful to note that the same enhancement of productivity also occurs in closed systems with dissipation, since the emptying is due to the dissipation instead of the flow out of the system. It is the competition between emptying and filling, not the mechanism of each, the essential features of the process that induces the singular behavior.

Secondly, a necessary requirement for a model is robustness under small perturbations. For chaotic systems, the outcome of the system is sensitively dependent on the initial conditions. In view of this, we considered a model to be robust if the sets of all possible outcomes

of the two slightly different versions of the model, say **A** and **B**, are very similar, where model **A**, $d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}, t)$; model **B**, $d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}, t) + \epsilon(t)$, where $\epsilon(t)$ is an arbitrarily small time dependent perturbation that is bounded. Successful modeling would require that the set of all possible outcomes from model **A** agrees closely with the set of all possible outcomes from model **B**. Difficulties appear when there are trajectories of **A** that do not closely follow *any* trajectory of **B** (or vice versa) for all but short periods of time, because, if trajectories from the closely related models do not agree, either model is presumably useless in representing the *physical system.* The hierarchy of difficulty levels that can obstruct modeling shadowability and hence impede the ability to model certain physical processes are as follows: (i) mild: simple chaos or sensitive dependence on initial conditions, (ii) moderate: nonhyperbolicity due to quadratic tangencies of stable and unstable manifolds, and (iii) severe: unstable dimension variability. For these systems, the modeling shadowability times are surprisingly short.

Thirdly, the incoming nonlinear communication signal is assumed to have been generated by an oscillator nearly identical to the synchronizing oscillator; we thus refer to our method as "dynamically matched filtering." A state estimate based on several Poincaré cycles of the incoming signal was used to derive a control signal that caused the synchronizing oscillator to track the incoming signal, but with reduced in-band noise. This method produced a lower-noise version of a chaotic signal containing an encoded symbolic message. If the transmitted signal passes through a noisy communication channel, then the dynamically matched filter could be used to produce a cleaner signal for retransmission. As the filter is dynamically matched to the signal, the time required for filtering and signal retransmission was very short. The potential technological application is promising since this kind of filter is expected to be simple, fast, and very efficient.

III. PROGRESS

The following papers have been written with this ONR grant support.

1. "Riddling of Chaotic Sets in Periodic Windows," Y. C. Lai and C. Grebogi, Phys. Rev. Lett. **83**, 2926–2929 (1999).

2. "Fractality, Chaos, and Reactions in Imperfectly Mixed Open Hydrodynamical Flows," Á. Péntek, G. Károyi, I. Scheuring, T. Tél, Z. Toroczkai, J. Kadtke, and C. Grebogi, Physica A **274**, 120–131 (1999); also Nato Proc. (1999).

3. "Unstable Dimension Variability in Coupled Chaotic Systems," Y. C. Lai, D. Lerner, K. Williams, and C. Grebogi, Phys. Rev. E **60**, 5445–5454 (1999).

4. "Obstruction to Deterministic Modeling of Chaotic Systems with Invariant Subspace," Y. C. Lai and C. Grebogi, Intern. J. Bif. and Chaos **10**, 683–693 (2000).

5. "Chaotic Advection, Diffusion, and Reactions in Imperfectly Mixed Open Hydrodynamical Flows," T. Tél, G. Károlyi, Á. Péntek, I. Scheuring, Z. Toroczkai, C. Grebogi, and J. Kadtke, Chaos **10**, 89–98 (2000).

6. "Feedback Synchronization Using Pole-placement Control," R. Tonelli, Y. C. Lai, and C. Grebogi, Intern. J. Bif. and Chaos, in print.

7. "Exploiting the Natural Redundancy of Chaotic Signals in Communication Systems," I. P. Mariño, E. Rosa, and C. Grebogi, Phys. Rev. Lett. **85**, 2629 (2000).

8. "Comunication Through Chaotic Modeling of Languages," M. Baptista, E. Rosa, and C. Grebogi, Phys. Rev. E **61**, 3590–3601 (2000).

9. "Driving Trajectories in Chaotic Systems," E. E. N. Macau and C. Grebogi, Intern. J. Bif. and Chaos, submitted.

10. "Topology of High-dimensional Chaotic Scattering," Y. C. Lai, A. Moura, and C. Grebogi, Phys. Rev. E, in print.

11. "Integrated Chaotic Communication Scheme," M. S. Baptista, E. E. N. Macau, C. Grebogi, Y. C. Lai, and E. Rosa, Phys. Rev. E **62** (2000).

12. "Signal Dropout Reconstruction in Communicating with Chaos," D. L. Valladares, S. Boccaletti, C. Grebogi, and H. Mancini, Intern. J. Bif. and Chaos, submitted.

13. "Unstable Dimension Variability and Synchronization of Chaotic Systems," R. L. Viana and C. Grebogi, Phys. Rev. E **62**, 462–468 (2000).

14. "Dynamics of a Mapping of the Type Hénon-Lozi," M. A. Aziz-Alaoui, C. Robert, and C. Grebogi, Chaos, Solitons and Fractals, in print.

15. "Necessity of Statistical Modeling of Deterministic Chaotic Systems," Y. C. Lai and C. Grebogi, in Tohwa Stat. Phys 99 (American Institute of Physics, 2000), pp. 531–542.

16. "Riddling in Dynamical Systems," Y. C. Lai and C. Grebogi, in *Equadiff 99*, Ed. B. Fiedler (World Scientific, 2000).

17. "Control of Chaos and Targeting," E. E. N. Macau and C. Grebogi, Proc. of the ICONNE Conference, Campos do Jordão, Brazil (2000).

18. "Autocatalytic Reactions of Active Particle with Phase," G. Santoboni, T. Nishikawa, Z. Toroczkai, and C. Grebogi, submitted for publication.

19. "Reconstruction of Information-bearing Chaotic Signals in Additive White Gaussian Noise: Performance Analysis and Evaluation," J. P. Mariño, C. Grebogi, and E. Rosa, submitted for publication.

20. "Integrated Chaotic Communication Scheme," M. S. Baptista, E. E. N. Macau, C. Grebogi, Y. -C. Lai, and E. Rosa, Physical Review E, to appear.

21. "Why Are Chaotic Attractors Rare in Multistable Systems?," U. Feudel and C. Grebogi, submitted for publication.